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NLO correction to one-particle inclusive production at high energies.

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Abstract

Next-to-leading order correction to the one-particle inclusive cross section in the framework of high energy factorization is calculated. Numerical results for midrapidity region are compared with predictions of conventional calculations based on collinear factorization.

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1 Introduction

The recently completed calculation of the NLO correction to BFKL Pomeron [1] involves as its ingredients the formulae for the cross section of tree level two-particle production and one-loop virtual correction to one-particle production in Quasi Multi Regge Kinematics (QMRK). These results can be used in studying the properties of the inclusive two-particle production at leading order [2, 3] and of the one-particle inclusive cross section in the next-to-leading order at high energies.

The total cross section in the next-to-leading order(NLO BFKL) was calculated in [1] and is now under an extensive discussion [4, 5]. The total cross section has a complicated structure due to delicate cancellations of singularities in various virtual contributions and infrared divergencies arising in integrations over parameters of real correction. Cancellation between divergent parts of real and virtual contributions is also needed for computation of the next-to-leading order one-particle inclusive production cross section, but here the structure of this cancellation is simpler and more transparent than in the case of total cross section.

The problem of one-particle inclusive production in high energy hadron collision was investigated, in the leading order (LO), in a number of works [3]. The purpose of this paper is to calculate one-particle inclusive cross section to the next-to-leading order at high energies proceeding as far as possible with analytical calculations and then turning to numerical estimates.

The outline of the paper is as follows.

In the second section we briefly review the results on the particle production cross sections obtained within the high energy factorization scheme and compare them to the expressions obtained using collinear factorization.

In the third section we describe a calculation leading to the explicit expression for the one-particle inclusive production cross section in the next-to-leading order.

In Section 4 some numerical results on particle production in central rapidity region are presented and discussed.

Section 5 contains a brief conclusion.

2 Particle production at high energies

From the theoretical viewpoint the hadron scattering at very high energies is special in the way the hard degrees of freedom (partons) are formed from colliding hadrons. When the ratio of hardness of the process $k_\perp$ (which is the transverse momentum of produced particle) to the invariant energy of colliding hadrons, $\sqrt{S}$, is not too small (up to 10^{-2}) it is possible to describe the structure functions of hadrons by taking into account only processes that contribute logarithms of $k_\perp/\Lambda_{QCD}$ at leading order, i.e. by resummation of the $\alpha_s^n \ln^n(k_\perp/\Lambda_{QCD})$ terms.

Such structure function is given DGLAP evolution equation [6].

Since the emergence of the combination $\alpha_s \ln(k_\perp/\Lambda_{QCD})$ implies strong ordering of emitted particles in their transverse momenta up until a hard collision block, the transverse momentum of detected particle $k_\perp$ is parametrically bigger than that of any parton involved in the process. Therefore it is possible to calculate the cross section of the hard process using the initial on-shell partons. This prescription is known as collinear factorization [7] and leads to the well-known result for the production rate:

$$\frac{d\sigma}{dk^2 dy_1} = 2\int dy_2 x_1 f_a(x_1, k^2)\frac{d\hat{\sigma}_{ab}}{dk^2} x_2 f_b(x_2, k^2), \tag{1}$$

where $f_a(x, k^2)$ is a structure function for the parton of type a and $x_{1,2} = k_\perp(e^{\pm y_1} + e^{\pm y_2})/\sqrt{S}$.

At high energies for particles produced with $k_\perp \ll \sqrt{S}$ another big logarithm, $\ln(1/x)$, is important. The resummation of such logarithmic contributions can become more important than of $\ln(k^2/\Lambda^2_{QCD})$. The resummation of the leading energy logarithms for the structure function is described by BFKL equation [8]. The domain of validity of the BFKL equation in describing the structure functions is at present not well understood. It is likely that for some kinematical region the correct approach is to resum the logarithms of both types, or at least interpolate between two types of resummation as done, e.g., in the CCFM[9] equation.

The main point is that at high energies the transverse momenta of the incoming parton fluxes can no longer be neglected. To take them into account a new approach called $k_\perp$ or high energy factorization was proposed [10, 11]. Extensive description of the method and various applications can be found in [11]. Let us note that this method was *de facto* used earlier in [12].

The method of high energy factorization is based on consideration of "partons" with nonzero transverse momentum that are, in contrast with the traditional collinear factorization case, virtual particles. In this case colliding hadrons are described by unintegrated structure function, ϕ, so that

$$\phi(x, q^2) = q^2 \frac{\partial x g(x, q^2)}{\partial q^2}, \tag{2}$$

where ϕ/q^2 is proportional to the probability to find the incident parton (essentially a gluon, because gluons give leading contribution at high energies) with the longitudinal momentum component xp_a (p_a is a momentum of the incident particle) and transverse momentum component $q_\perp$.

Note that such an interpretation is somewhat oversimplified, because due to the quantum structure of QCD evolution some form-factors may arise changing the form of ϕ. For DGLAP evolution it is a Sudakov form-factor [13]. However, when studying the semi-inclusive quantity like one-particle production cross section it is legitimate to use the unintegrated structure function in the simple form

of Eq.(2)[2].

Scattering of the off-shell "partons" are described by generalized cross sections calculated in Quasi Multi Regge Kinematics (QMRK) [14, 15].

In QMRK one studies the $2 \to n+2$ scattering process with two outgoing particles having almost the same momenta as the incident ones and remaining n particles emitted into the central rapidity region separated by large rapidity gaps from incoming particles (the situation, where rapidity gaps between n particles are also large, corresponds to Multi Regge Kinematics, MRK). Large rapidity gaps allow to distinguish the quantities related to the incident particles from those describing the cross section of the hard process of particle production in the central rapidity region.

Let us for example consider the cross section of the process $gg \to ggg$ in the limit of high energy:

$$\frac{d\sigma_{gg\to ggg}}{d^2k_\perp dy} = \frac{4N_c^3\alpha_s^3}{\pi^2(N_c^2-1)}\int \frac{d^2q_{1\perp}}{q_{1\perp}^2}\frac{\delta^{(2)}(q_{1\perp}+q_{2\perp}-k_\perp)}{k_\perp^2}\frac{d^2q_{2\perp}}{q_{2\perp}^2}, \tag{3}$$

where $q_{1,2} = p_{a,b} - p'_{a,b}$. in Eq. (3) the above-mentioned factorization of the cross section is clearly seen. Indeed, the first, second and third factors under the integral correspond to $p_a \to p'_a, q_1$ splitting, $q_1, q_2 \to k$ "scattering" and $p_b \to p'_2, q_2$ splitting respectively.

The factors related to the splitting of the incident particles should further be transformed to structure functions. This can be done in two steps. First, one assembles incident partons into wave packets describing by form factors [2]. The second step is taking into account additional radiation and virtual corrections summed to give the unintegrated structure functions $\varphi(x, q_\perp)$ with x fixed by the kinematics of the considered process.

The cross sections of producing $n = 1, 2$ particles in the central region to the lowest perturbative order read:

$$\begin{aligned}\frac{d\sigma_1}{d^2k_\perp dy} &= \int d^2q_{1\perp}d^2q_{2\perp}\frac{\varphi(x_{1,0},q_{1\perp})}{q_{1\perp}^2}\frac{d\hat\sigma_1}{dk_\perp^2}\frac{\varphi(x_{2,0},q_{2\perp})}{q_{2\perp}^2},\\ \frac{d\hat\sigma_1}{dk_\perp^2} &= \frac{4N_c\alpha_s}{N_c^2-1}\frac{\delta^{(2)}(q_{1\perp}+q_{2\perp}-k_\perp)}{k_\perp^2},\\ x_{1,0} &= k_\perp e^y/\sqrt{S}, \quad x_{2,0} = k_\perp e^{-y}/\sqrt{S};\end{aligned} \tag{4}$$

$$\begin{aligned}\frac{d\sigma_2}{d^2k_{1\perp}d^2k_{2\perp}dy_1dy_2} &= \int d^2q_{1\perp}d^2q_{2\perp}\frac{\varphi(x_1,q_{1\perp})}{q_{1\perp}^2}\frac{d\hat\sigma_2}{d^2k_{1\perp}d^2k_{2\perp}d\Delta y}\frac{\varphi(x_2,q_{2\perp})}{q_{2\perp}^2},\\ \frac{d\hat\sigma_2}{d^2k_{1\perp}d^2k_{2\perp}d\Delta y} &= \frac{2N_c^2\alpha_s^2}{(N_c^2-1)\pi^2}\frac{\delta^{(2)}(q_{1\perp}+q_{2\perp}-k_{1\perp}-k_{2\perp})}{q_{1\perp}^2q_{2\perp}^2}\mathcal{A},\end{aligned} \tag{5}$$

[2]I am grateful to Yu.L.Dokshitzer for pointing me this out

$$x_1 = k_{1\perp} e^{y_1}(1 + k_{2\perp} e^{\Delta y})/\sqrt{S},\ x_2 = k_{1\perp} e^{-y_1}(1 + k_{2\perp} e^{-\Delta y})/\sqrt{S}.$$
$$\Delta y = y_2 - y_1.$$

A part of the analytical expression for $\mathcal{A}$ can be found in [15, 16] for the subprocesses $gg \to gg$ and in [15, 17] for the $gg \to q\bar{q}$ ones. The explicit form of $\mathcal{A}$ was recently derived in [18] and is given in Appendix A. Note that the formula for $gg \to q\bar{q}$ cross section from [17] coincides with the analogous formula in [11] in the limit of massless quarks.

Eq.(4) gives the rate of one-particle production in the leading order. It was studied in a number of publications [3]. Our aim is to calculate first correction to it.

3 Real and virtual contributions to NLO one-particle production

The one-particle production in the next-to-leading order includes two contributions, real and virtual. The real contribution comes from the two-particle cross section Eq.(5) integrated over the phase space of one of the particles (considered unobservable) and the fixed four-momentum of the second particle:

$$\frac{d\sigma_r}{d^2k_{1\perp}dy_1} = 2\int d\Phi \frac{\varphi(x_1, q_{1\perp})}{q_{1\perp}^2} \frac{d\hat{\sigma}_2}{d^2k_{1\perp}d^2k_{2\perp}d\Delta y} \frac{\varphi(x_2, q_{2\perp})}{q_{2\perp}^2} \tag{6}$$

where

$$d\Phi = d^2q_{1\perp}d^2q_{2\perp}d^2k_{2\perp}d\Delta y \tag{7}$$

The factor of 2 in Eq. (6) reflects the identity of outgoing particles.

The virtual contribution has the same form as in Eq.(4), but instead of $\hat{\sigma}_1$ we must use

$$\frac{d\hat{\sigma}_v}{dk_\perp^2} = \frac{4N_c\alpha_s}{N_c^2 - 1} \frac{\delta^{(2)}(q_{1\perp} + q_{2\perp} - k_\perp)}{k_\perp^2} \mathcal{V}(q_{1\perp}, q_{2\perp}) \tag{8}$$

The virtual correction contains both ultraviolet and infrared divergencies. The ultraviolet one leads, through standard renormalization procedure, to the running coupling constant. The infrared divergence must cancel with the infrared and collinear divergencies in the real contribution.

3.1 Cancellation of collinear and infrared divergencies

Let us now outline at the formal level how this cancellation occurs. To deal with the collinear singularity we must introduce a jet defining algorithm which in two particle production case could be expressed through the function $S(k, k_1, k_2)$ (k_1, k_2 and k are on-shell 4-vectors) so that Eq.(6) is replaced by

$$\frac{d\sigma_r}{d^2k_\perp dy} = \int d^2k_{1\perp}d^2k_{2\perp}dy_1dy_2 \frac{d\sigma_2}{d^2k_{1\perp}d^2k_{2\perp}dy_1dy_2} S(k, k_1, k_2). \tag{9}$$

To provide the sought for cancellation between the real and virtual corrections S should be an infrared safe quantity, which means that the following property should hold (cf. [19]):

$$S(k, \lambda k_1, (1-\lambda)k_1) = \delta^{(2)}(k_\perp - k_{1\perp})\delta(y - y_1), \quad 0 < \lambda < 1 \tag{10}$$

In the following we choose S in the following form:

$$\begin{aligned} S(k, k_1, k_2) &= \theta(R > R_0) \sum_{i=1,2} \delta^{(2)}(k_\perp - k_{i\perp})\delta(y - y_i) \\ &+ \theta(R < R_0)\delta^{(2)}(k_\perp - k_{1\perp} - k_{2\perp})\delta\left(y - \frac{1}{2}\ln\frac{k_{1\perp}e^{y_1} + k_{2\perp}e^{y_2}}{k_{1\perp}e^{-y_1} + k_{2\perp}e^{-y_2}}\right), \end{aligned} \tag{11}$$

where $R^2 = (\phi_1 - \phi_2)^2 + (y_1 - y_2)^2$. Although the last line in Eq.(11) may look artificial, it has the natural meaning. If we claim that "combined" particle, formed by two particles indistinguishable under given resolution, has definite rapidity and that 4-momenta of two particles are added up to form the 4-momentum of "combined" particle, we immediately arrive at Eq.(11). It is straightforward to check that Eq.(11) satisfies Eq. (10).

Grouping together Eqs.(5),(9) and (11) we find:

$$\begin{aligned} \frac{d\sigma_r}{d^2k_\perp dy} &= 2\int d\Phi \frac{\varphi(x_1, q_{1\perp})}{q_{1\perp}^2} \frac{d\hat{\sigma}_2}{d^2k_\perp d^2k_{2\perp} d\Delta y} \frac{\varphi(x_2, q_{2\perp})}{q_{2\perp}^2} \theta(R > R_0) \\ &+ \int d\Phi \frac{\varphi(\tilde{x}_1, q_{1\perp})}{q_{1\perp}^2} \frac{d\hat{\sigma}_2}{d^2k_{1\perp} d^2k_{2\perp} d\Delta y} \frac{\varphi(\tilde{x}_2, q_{2\perp})}{q_{2\perp}^2} \theta(R < R_0), \end{aligned} \tag{12}$$

where $k_{1\perp} = k_\perp - k_{2\perp}$ and $\tilde{x}_{1,2} = e^{\pm y}\sqrt{\Sigma/S}$ with $\Sigma = k_1^2 + k_2^2 + 2k_1k_2\mathrm{ch}(\Delta y)$ (see Appendix A), and $k_i = |k_{i\perp}|$ which is a notation we shall use from now on. Note that when $R \to 0$ $\tilde{x}_1 \to x_{1,0}$ and $\tilde{x}_2 \to x_{2,0}$. Let us now rewrite the second term in Eq.(12) as:

$$\begin{aligned} &\left[\int d\Phi \left(\frac{\varphi(\tilde{x}_1, q_{1\perp})}{q_{1\perp}^2}\frac{\varphi(\tilde{x}_2, q_{2\perp})}{q_{2\perp}^2} - \frac{\varphi(x_{1,0}, q_{1\perp})}{q_{1\perp}^2}\frac{\varphi(x_{2,0}, q_{2\perp})}{q_{2\perp}^2}\right) \frac{d\hat{\sigma}_2}{d^2k_{1\perp} d^2k_{2\perp} d\Delta y} \right. \\ &\left. + \int d\Phi \frac{\varphi(x_{1,0}, q_{1\perp})}{q_{1\perp}^2} \frac{d\hat{\sigma}_2}{d^2k_{1\perp} d^2k_{2\perp} d\Delta y} \frac{\varphi(x_{2,0}, q_{2\perp})}{q_{2\perp}^2}\right] \theta(R < R_0). \end{aligned} \tag{13}$$

The integration in the first line is free from divergencies, while in the second line integrals over $d^2k_{2\perp}$ and $d\Delta y$ do not involve structure functions and could be done analytically. Before doing this integration we make a replacement $\theta(R < R_0) = 1 - \theta(R > R_0)$ in the last term and then substitute Eq.(13) to Eq.(12) to yield:

$$\frac{d\sigma_r}{d^2k_\perp dy} =$$

$$
\begin{aligned}
&\int d^2q_{1\perp}d^2q_{2\perp}\frac{\varphi(x_{1,0},q_{1\perp})}{q_{1\perp}^2}\frac{\varphi(x_{2,0},q_{2\perp})}{q_{2\perp}^2}\int d^2k_{2\perp}d\Delta y\frac{d\hat{\sigma}_2}{d^2k_{1\perp}d^2k_{2\perp}d\Delta y}\\
&+\int d\Phi\left(2\frac{\varphi(x_1,q_{1\perp})}{q_{1\perp}^2}\frac{d\hat{\sigma}_2}{d^2k_{\perp}d^2k_{2\perp}d\Delta y}\frac{\varphi(x_2,q_{2\perp})}{q_{2\perp}^2}\theta(R(k,k_2)>R_0)\right.\\
&\qquad\left.-\frac{\varphi(x_{1,0},q_{1\perp})}{q_{1\perp}^2}\frac{d\hat{\sigma}_2}{d^2k_{1\perp}d^2k_{2\perp}d\Delta y}\frac{\varphi(x_{2,0},q_{2\perp})}{q_{2\perp}^2}\theta(R(k_1,k_2)>R_0)\right)\\
&+\int d\Phi\left(\frac{\varphi(\tilde{x}_1,q_{1\perp})}{q_{1\perp}^2}\frac{\varphi(\tilde{x}_2,q_{2\perp})}{q_{2\perp}^2}-\frac{\varphi(x_{1,0},q_{1\perp})}{q_{1\perp}^2}\frac{\varphi(x_{2,0},q_{2\perp})}{q_{2\perp}^2}\right)\\
&\qquad\frac{d\hat{\sigma}_2}{d^2k_{1\perp}d^2k_{2\perp}d\Delta y}\theta(R<R_0),
\end{aligned}
\tag{14}
$$

where we indicate that $R(k,k_2)=((y-y_2)^2+(\phi-\phi_2)^2)^{1/2}$ and $R(k_1,k_2)=((y_1-y_2)^2+(\phi_1-\phi_2)^2)^{1/2}$ are different in the third and forth lines of Eq.(14). The third line in Eq.(14) has an infrared singularity when $k_2\to 0$ and the forth one has singularities when $k_2\to 0$ or $k_1\to 0$. However, it is possible to combine the singularities in the forth line so that we find only one singular point and also an accompanying factor of two thus providing a cancellation of singularities in the third and forth lines.

Indeed, the expression under the integral in the forth line in Eq.(14) is symmetric under the simultaneous transformation $k_{1\perp}\leftrightarrow k_{2\perp}$ and $\Delta y\leftrightarrow -\Delta y$ which is nothing else than the permutation of the two produced particles. Therefore we can simply multiply this term at $2\theta(k_{1\perp}>k_{2\perp})$ to obtain:

$$
\begin{aligned}
&\frac{d\sigma_r}{d^2k_\perp dy}=\\
&\int d^2q_{1\perp}d^2q_{2\perp}\frac{\varphi(x_{1,0},q_{1\perp})}{q_{1\perp}^2}\frac{\varphi(x_{2,0},q_{2\perp})}{q_{2\perp}^2}\int d^2k_{2\perp}d\Delta y\frac{d\hat{\sigma}_2}{d^2k_{1\perp}d^2k_{2\perp}d\Delta y}\\
&+2\int d\Phi\left(\frac{\varphi(x_1,q_{1\perp})}{q_{1\perp}^2}\frac{d\hat{\sigma}_2}{d^2k_{\perp}d^2k_{2\perp}d\Delta y}\frac{\varphi(x_2,q_{2\perp})}{q_{2\perp}^2}\theta(R(k,k_2)>R_0)\right.\\
&\quad\left.-\frac{\varphi(x_{1,0},q_{1\perp})}{q_{1\perp}^2}\frac{d\hat{\sigma}_2}{d^2k_{1\perp}d^2k_{2\perp}d\Delta y}\frac{\varphi(x_{2,0},q_{2\perp})}{q_{2\perp}^2}\theta(R(k_1,k_2)>R_0)\theta(k_{1\perp}>k_{2\perp})\right)\\
&+\int d\Phi\left(\frac{\varphi(\tilde{x}_1,q_{1\perp})}{q_{1\perp}^2}\frac{\varphi(\tilde{x}_2,q_{2\perp})}{q_{2\perp}^2}-\frac{\varphi(x_{1,0},q_{1\perp})}{q_{1\perp}^2}\frac{\varphi(x_{2,0},q_{2\perp})}{q_{2\perp}^2}\right)\\
&\qquad\frac{d\hat{\sigma}_2}{d^2k_{1\perp}d^2k_{2\perp}d\Delta y}\theta(R<R_0).
\end{aligned}
\tag{15}
$$

The combination of the third and forth lines in Eq.(15) is free from singularities (note that for $k_2\to 0$ there is no difference between $R(k,k_2)$ and $R(k_1,k_2)$) and the singularity in the second line cancels with that in $\mathcal{V}$ in Eq.(8). Combining Eq.(15) with Eqs.(4),(8) we obtain the second order correction to the one-particle

inclusive production in the high energy factorization scheme:

$$
\begin{aligned}
&\frac{d\sigma^{(2)}}{d^2k_\perp dy} = \\
&\int d^2q_{1\perp}d^2q_{2\perp}\frac{\varphi(x_{1,0},q_{1\perp})}{q_{1\perp}^2}\frac{\varphi(x_{2,0},q_{2\perp})}{q_{2\perp}^2}\left(\frac{d\hat{\sigma}_2}{d^2k_\perp}+\frac{d\hat{\sigma}_v}{d^2k_\perp}\right) \\
&+2\int d\Phi\left(\frac{\varphi(x_1,q_{1\perp})}{q_{1\perp}^2}\frac{d\hat{\sigma}_2}{d^2k_\perp d^2k_{2\perp}d\Delta y}\frac{\varphi(x_2,q_{2\perp})}{q_{2\perp}^2}\theta(R(k,k_2)>R_0)\right. \qquad (16)\\
&-\left.\frac{\varphi(x_{1,0},q_{1\perp})}{q_{1\perp}^2}\frac{d\hat{\sigma}_2}{d^2k_{1\perp}d^2k_{2\perp}d\Delta y}\frac{\varphi(x_{2,0},q_{2\perp})}{q_{2\perp}^2}\theta(R(k_1,k_2)>R_0)\theta(k_{1\perp}>k_{2\perp})\right) \\
&+\int d\Phi\left(\frac{\varphi(\tilde{x}_1,q_{1\perp})}{q_{1\perp}^2}\frac{\varphi(\tilde{x}_2,q_{2\perp})}{q_{2\perp}^2}-\frac{\varphi(x_{1,0},q_{1\perp})}{q_{1\perp}^2}\frac{\varphi(x_{2,0},q_{2\perp})}{q_{2\perp}^2}\right) \\
&\qquad\qquad\frac{d\hat{\sigma}_2}{d^2k_{1\perp}d^2k_{2\perp}d\Delta y}\theta(R<R_0).
\end{aligned}
$$

Let us now analyse the MRK limit of Eq.(16), that is we take limit $\Delta y\to\infty$ in $\mathcal{A}$ (see Eq.(5) and Eq.(A.2) in Appendix). In this limit

$$
\mathcal{A}\to\mathcal{A}_{MRK}=\frac{q_{1\perp}^2q_{2\perp}^2}{k_{1\perp}^2k_{2\perp}^2} \qquad (17)
$$

which is precisely the combination of two leading order BFKL kernels responsible for real particles production. However, let us remind that the leading order one-particle production in high-energy factorization, described by Eq.(4) includes MRK contributions to all orders if the unintegrated structure function, $\phi(x,q_\perp)$ includes resummation to all orders of $\alpha_s\ln(1/x)$. It is evidently the case for structure functions undergoing BFKL equation. For other types of structure functions we just assume that the resummed $\alpha_s^n\ln^n(1/x)$ terms are included in some hidden way. Consequently, we must subtract $\mathcal{A}_{MRK}$ from $\mathcal{A}$, and we will imply this subtraction in the following.

To proceed further we must calculate

$$
\frac{d\hat{\sigma}_2}{d^2k_\perp}=\int d^2k_{2\perp}d\Delta y\frac{d\hat{\sigma}_2}{d^2k_{1\perp}d^2k_{2\perp}d\Delta y}. \qquad (18)
$$

Note that the quantity in brackets in the first line of Eq.(16) should coincide (up to the constant factors depending on normalization) with the NLO BFKL kernel written explicitly in [1]. Note however, that when calculating the real contribution in [16] the terms vanishing after integration over $d^2k_\perp$ were dropped which did not change the result for NLO BFKL Pomeron itself. In calculating the one-particle inclusive cross sections these contributions have to be kept.

The integration in Eq.(18) is very difficult. Fortunately, we can do it not for whole $\hat{\sigma}_2$, but only for its singular part $\hat{\sigma}_2^s$. We also have to change some other

terms in Eq.(16) that emerge when arriving from Eq.(12) at Eq.(16). Finally, the result is

$$
\begin{aligned}
&\frac{d\sigma^{(2)}}{d^2k_\perp dy} = \\
&\int d^2q_{1\perp}d^2q_{2\perp}\frac{\varphi(x_{1,0},q_{1\perp})}{q_{1\perp}^2}\frac{\varphi(x_{2,0},q_{2\perp})}{q_{2\perp}^2}\left(\frac{d\hat{\sigma}_2^s}{d^2k_\perp}+\frac{d\hat{\sigma}_v}{d^2k_\perp}\right) \\
&+2\int d\Phi\left(\frac{\varphi(x_1,q_{1\perp})}{q_{1\perp}^2}\frac{d\hat{\sigma}_2}{d^2k_\perp d^2k_{2\perp}d\Delta y}\frac{\varphi(x_2,q_{2\perp})}{q_{2\perp}^2}\theta(R(k,k_2)>R_0)\right. \qquad (19)\\
&-\left.\frac{\varphi(x_{1,0},q_{1\perp})}{q_{1\perp}^2}\frac{d\hat{\sigma}_2^s}{d^2k_{1\perp}d^2k_{2\perp}d\Delta y}\frac{\varphi(x_{2,0},q_{2\perp})}{q_{2\perp}^2}\theta(R(k_1,k_2)>R_0)\theta(k_{1\perp}>k_{2\perp})\right) \\
&+\int d\Phi\left(\frac{\varphi(\tilde{x}_1,q_{1\perp})}{q_{1\perp}^2}\frac{d\hat{\sigma}_2}{d^2k_{1\perp}d^2k_{2\perp}d\Delta y}\frac{\varphi(\tilde{x}_2,q_{2\perp})}{q_{2\perp}^2}\right. \\
&\qquad -\left.\frac{\varphi(x_{1,0},q_{1\perp})}{q_{1\perp}^2}\frac{d\hat{\sigma}_2^s}{d^2k_{1\perp}d^2k_{2\perp}d\Delta y}\frac{\varphi(x_{2,0},q_{2\perp})}{q_{2\perp}^2}\right)\theta(R<R_0).
\end{aligned}
$$

3.2 Integration.

Let us now choose the singular part of $\mathcal{A}$ with contributions from quark and gluon production added up in the form (see Appendix, MRK part as it was mentioned above is subtracted):

$$
\mathcal{A}^s = -\frac{q_1^2q_2^2}{2k_1^2k_2^2}+\frac{q_1^2q_2^2\mathrm{ch}(\Delta y)}{k_1k_2s}-\left(1-\frac{n_f}{4N_c}\right)\frac{2q_1^2q_2^2}{s\Sigma}+\left(\frac{D-2}{2}-\frac{n_f}{N_c}\right)\frac{E^2}{8s^2} \qquad (20)
$$

This form of $\mathcal{A}^s$ is chosen to avoid artificial ultraviolet divergency which occurs if one takes $\Sigma = k^2$ as it is in collinear and infrared limits. For the same purpose we take E in the form:

$$
E = \frac{1}{\Sigma}[k^2(\bar{q}_1-\bar{q}_2)(\bar{k}_1-\bar{k}_2)-(q_1^2-q_2^2)(k_1^2-k_2^2)+2k_1k_2\mathrm{sh}(\Delta y)(k^2-q_1^2-q_2^2)] \quad (21)
$$

Now we integrate the singular part of $\mathcal{A}$ over transverse two-dimensional momentum space analytically continued to $D-2 = 2+2\varepsilon$ or, strictly speaking, with

$$
d^2k_{2\perp} \to \frac{d^{2+2\varepsilon}k_{2\perp}}{(2\pi)^{2\varepsilon}} \qquad (22)
$$

and over rapidity. The results of the calculation of the integral over $\mathcal{A}^s$ are given in Appendix B. The answer reads (see Eq.(5) for relation between $\hat{\sigma}_2$ and $\mathcal{A}$)

$$
\begin{aligned}
\frac{d\hat{\sigma}_r^s}{d^2k_\perp} &= \frac{2N_c^2\alpha_s^2}{N_c^2-1}\frac{\delta^{(2)}(q_{1\perp}+q_{2\perp}-k_\perp)}{k^2}\frac{\Gamma(1-\varepsilon)}{(4\pi)^{1+\varepsilon}}\frac{4\Gamma^2(1+\varepsilon)}{\varepsilon\Gamma(1+2\varepsilon)}\left(\frac{k^2}{\mu^2}\right)^\varepsilon \qquad (23)\\
&\left(\frac{1}{\varepsilon}+2\psi(1)-2\psi(1+2\varepsilon)-\frac{11+8\varepsilon}{2(1+2\varepsilon)(3+2\varepsilon)}+\frac{n_f}{4N_c}\frac{4+6\varepsilon}{(1+2\varepsilon)(3+2\varepsilon)}\right)
\end{aligned}
$$

The result for $\hat{\sigma}_v$ was derived in [20]. Note that the answer depends on the arrangement of different corrections to QMRK amplitude. In this paper the symmetric variant [1] is chosen.

$$\frac{d\hat{\sigma}_v}{d^2k_\perp} = \frac{4N_c^2\alpha_s^2}{N_c^2-1}\frac{\delta^{(2)}(q_{1\perp}+q_{2\perp}-k_\perp)}{k^2}\frac{\Gamma(1-\varepsilon)}{(4\pi)^{1+\varepsilon}}\times$$
$$\left[-\frac{2}{\varepsilon^2}\left(\frac{k^2}{\mu^2}\right)^\varepsilon + \frac{1}{\varepsilon}\left(\frac{11}{3}-\frac{2}{3}\frac{n_f}{N_c}\right)+\pi^2+\frac{k^2}{3}\left\{\left(11-2\frac{n_f}{N_c}\right)\frac{\ln q_1^2/q_2^2}{q_1^2-q_2^2}+\right.\right. \tag{24}$$
$$\left.\left.\left(1-\frac{n_f}{N_c}\right)\left(\left(\frac{q_1^2}{q_2^2}-\frac{q_2^2}{q_1^2}-2\ln\frac{q_1^2}{q_2^2}\right)\frac{2q_1^2q_2^2-\bar{q}_1\bar{q}_2(q_1^2+q_2^2+4\bar{q}_1\bar{q}_2)}{(q_1^2-q_2^2)^3}+\frac{\bar{q}_1\bar{q}_2}{q_1^2q_2^2}\right)\right\}\right]$$

From Eqs.(23) and (24) it is easy to see that divergencies of real and virtual parts cancel leaving a finite contribution to the first line in Eq.(19):

$$\frac{d\hat{\sigma}_r^s}{d^2k_\perp}+\frac{d\hat{\sigma}_v}{d^2k_\perp} = \frac{N_c^2\alpha_s^2}{(N_c^2-1)\pi}\frac{\delta^{(2)}(q_{1\perp}+q_{2\perp}-k_\perp)}{k^2}\left[-\left(\frac{11}{3}-\frac{2}{3}\frac{n_f}{N_c}\right)\ln\frac{k^2}{\mu^2}\right.$$
$$-\frac{2\pi^2}{3}+\frac{64}{9}-\frac{7}{9}\frac{n_f}{N_c}+\frac{k^2}{3}\left\{\left(11-2\frac{n_f}{N_c}\right)\frac{\ln q_1^2/q_2^2}{q_1^2-q_2^2}+\right. \tag{25}$$
$$\left.\left.\left(1-\frac{n_f}{N_c}\right)\left(\left(\frac{q_1^2}{q_2^2}-\frac{q_2^2}{q_1^2}-2\ln\frac{q_1^2}{q_2^2}\right)\frac{2q_1^2q_2^2-\bar{q}_1\bar{q}_2(q_1^2+q_2^2+4\bar{q}_1\bar{q}_2)}{(q_1^2-q_2^2)^3}+\frac{\bar{q}_1\bar{q}_2}{q_1^2q_2^2}\right)\right\}\right]$$

The first term in Eq.(25), which is proportional to $\ln(k^2/\mu^2)$, is nothing but the well-known contribution corresponding to the running coupling. Therefore, after replacing α_s by the running coupling $\alpha_s(k^2)$, one should drop this term.

The Eqs. (19) and (25) together with Eq.(20) and the formula from Appendix A provide an analytical expression for the one-particle inclusive cross section.

For practical applications one should integrate over the parameters of unintegrated structure functions (disregarding trivial elimination of delta-functions). The corresponding numerical calculations numerical studies will be descried in the next Section.

4 Numerical estimates

The numerical results strongly depend on the type of structure functions used in the calculation. Let us first consider the asymptotic BFKL structure function [21]

$$\phi(x,q^2) = C\left(\frac{x_0}{x}\right)^\lambda \frac{q}{q_0}\frac{1}{\sqrt{\pi\lambda''\ln(x_0/x)}}\exp\left[-\frac{\ln^2(q^2/q_0^2)}{4\lambda''\ln(x_0/x)}\right], \tag{26}$$

with $\lambda = 4\ln 2N_c\alpha_s/\pi$ and $\lambda'' = 14\zeta(3)N_c\alpha_s/\pi$; α_s is chosen to be equal 0.2. Because the asymptotic BFKL structure function is a solution of the linear homogeneous equation, it does not have definite normalization and, moreover, the

parameters q_0 and x_0 are arbitrary. Since the calculation with this structure function is illustrative only, we simply choose $C = 1$, $q_0 = 1\mathrm{GeV}$ and $x_0 = 1$. From Eq.(19) it is clear that the NLO correction to the production process depends on the parameter R describing the collinear angle. For this calculation we take $R = 0.7$. Furhter discussion of the R dependence will be given below.

It is important to note that although in asymptotic BFKL structure function the strong coupling constant does not run, to be consistent we should however make it run in a semihard vertices (cf. the discussion after Eq.(19)). In the actual calculation with 1-loop α_s we choose $\Lambda_{QCD} = 200\mathrm{MeV}$.

The one-particle inclusive cross section for $\sqrt{S} = 5.5\mathrm{TeV}$, $y = 0$, and $n_f = 4$ is shown in Fig. 1 where for collinear factorization Eq.(1) was used with

$$xg(x, k^2) = \int\limits_0^{k^2} \frac{dq^2}{q^2}\phi(x, q^2)$$

As our second example we study the production process using the AKMS structure function [22]. In AKMS approach unintegrated structure function is obtained by application of BFKL equation to the evolution of structure function on x from $x = 10^{-2}$ where $\phi(10^{-2}, k^2)$ is chosen to give the best fit for experimental data.

This structure function can be fitted with an acceptable accuracy at least in the range of interest $x = 10^{-2}..10^{-4}$ by

$$\phi(x, q^2) = \frac{A}{x^\beta}\frac{q}{q_0}\exp\left[-\frac{B(x)\ln^2(q^2/q'^2(x))}{4\ln(1/x)}\right], \tag{27}$$

with parameters $A = 8.55 \cdot 10^{-2}$, $\beta = 0.486$, $q' = 0.758 + 5.41x^{0.6816}$ and $B = 3.51 - 2.91\ln\ln(1/x) + 0.793(\ln\ln(1/x))^2$. The results of calculation are given in Fig. 2 again with 1-loop α_s, $\Lambda_{QCD} = 200\mathrm{MeV}$, $\sqrt{S} = 5.5\mathrm{TeV}$, $n_f = 4$, $y = 0$, and $R = 0.7$.

Finally, in Fig. 3 we show the one-particle inclusive cross sections calculated with GRV94(NLO) structure function [23] satisfying the DGLAP equation not related to BFKL. However, it possibly includes leading MRK contribution through the initial conditions for DGLAP evolution. Note the rapid increase and broadening of structure function with decreasing x, which is the characteristic property of BFKL induced structure functions. This calculation was done for $\sqrt{S} = 1.8, 5.5$ and $14\mathrm{TeV}$, 2-loop α_s with $\Lambda_{QCD} = 200\mathrm{MeV}$ (because with these parameters GRV94(NLO) is calculated), $n_f = 4$, $y = 0$, and $R = 0.7$.

From Figs. 1,2,3 we see that taking into account the NLO corrections leads to the decrease of the particle production rate at high energies. For the structure functions obtained in BFKL approach NLO corrections change cross sections substantially (up to 50% in chosen kinematical interval), for non-BFKL GRV structure function changes are more dramatic: corrected cross sections are 2 to

5 times smaller than the leading order ones (apparently this results are sensitive to the cone size). We have no explanation to this fact. We hope that further studies of relationships between different types of structure functions may make this subject clearer.

Figs. 3a,b,c show that ratios of differential cross sections calculated to the LO and NLO accuracy in high energy factorization and to the LO in collinear factorization are insensitive to the c.m.s. energy in the domain $x \ll 1$.

Finally, in Fig. 4 we show the dependence of NLO cross section on the cone size R. It may be fitted well by function of the type $A+B\ln R+CR$ and becomes infinitely large (and negative) at $R \to 0$. This is the general property of quantities with cancelling virtual and real corrections showing that at small values of R the fixed order perturbation theory is not valid (cf. [24]).

5 Conclusion

The paper is devoted to the calculation of next-to-leading order correction to one particle inclusive production in the framework of high-energy factorization. High energy factorization scheme allows one to account for the initial transverse momentum of the colliding partons. The natural setup for particle production processes leading to high energy factorization is provided by Quasi-Multy Regge Kinematics.

The results of computation of NLO contributions to BFKL Pomeron (cf.[1] and references therein) can be used to compute the next-to-leading order corrections to one particle inclusive production at high energies. This correction includes real and virtual pieces. The infrared singularity in the virtual piece in the NLO contribution cancels the infrared singularity in its real one when an infrared safe jet algorithm is applied. The explicit calculations of the infrared stable one-particle inclusive cross section at the next-to-leading order constitutes the main result of the paper.

Numerical estimates were made to analyze the magnitude of NLO corrections for typical semihard transverse momenta and central rapidity region. Here we observe an essential dependence on the type of the structure functions used in the computation and shows more stability for BFKL-type structure functions and than for DGLAP one.

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Appendix A Cross sections of pair production in high energy factorization

We will use the following notation:

$$s = 2(k_1 k_2 \mathrm{ch}(\Delta y) - k_{1\perp} k_{2\perp});$$

$$t = -(q_{1\perp} - k_{1\perp})^2 - k_1 k_2 e^{\Delta y},\ u = -(q_{1\perp} - k_{2\perp})^2 - k_1 k_2 e^{-\Delta y};$$

$$\Sigma = x_1 x_2 S = k_1^2 + k_2^2 + 2k_1 k_2 \mathrm{ch}(\Delta y),$$

with $k_1 = \sqrt{k_{1\perp}^2}$, $k_2 = \sqrt{k_{2\perp}^2}$ and $k_{1\perp} k_{2\perp}$ is the dot product with 2d Euclidean metric.

Combined gluons and quarks (fermions) contribution to gg scattering has the form (adopted to (5))

$$\mathcal{A} = \mathcal{A}_{gluons} + \frac{n_f}{4N_c^3} \mathcal{A}_{fermions} \tag{A.1}$$

A.1 $gg \to gg$

$$\mathcal{A}_{gluons} = \mathcal{A}_1 + \mathcal{A}_2$$

$$\begin{aligned} \mathcal{A}_1 = \quad q_1^2 q_2^2 \quad & \Bigg\{ -\frac{1}{tu} + \frac{1}{4tu} \frac{q_1^2 q_2^2}{k_1^2 k_2^2} - \frac{e^{\Delta y}}{4tk_1 k_2} - \frac{e^{-\Delta y}}{4uk_1 k_2} + \frac{1}{4k_1^2 k_2^2} + \\ & \frac{1}{\Sigma} \Bigg[-\frac{2}{s} \left(1 + k_1 k_2 (\frac{1}{t} - \frac{1}{u}) \mathrm{sh}(\Delta y) \right) + \frac{1}{2k_1 k_2} (1 + \frac{\Sigma}{s}) \mathrm{ch}(\Delta y) - \\ & - \frac{q_1^2}{4s} [(1 + \frac{k_2}{k_1} e^{-\Delta y}) \frac{1}{t} + (1 + \frac{k_1}{k_2} e^{\Delta y}) \frac{1}{u}] \\ & - \frac{q_2^2}{4s} [(1 + \frac{k_1}{k_2} e^{-\Delta y}) \frac{1}{t} + (1 + \frac{k_2}{k_1} e^{\Delta y}) \frac{1}{u}] \Bigg] \Bigg\} \end{aligned} \tag{A.2}$$

$$\mathcal{A}_2 = \frac{D-2}{4} \left\{ \left(\frac{(k_{1\perp} - q_{1\perp})^2 (k_{2\perp} - q_{1\perp})^2 - k_1^2 k_2^2}{tu} \right)^2 - \right.$$

$$\left. -\frac{1}{4} \left(\frac{(k_{2\perp} - q_{1\perp})^2 - k_1 k_2 e^{-\Delta y}}{(k_{2\perp} - q_{1\perp})^2 + k_1 k_2 e^{-\Delta y}} - \frac{E}{s} \right) \left(\frac{(k_{1\perp} - q_{1\perp})^2 - k_1 k_2 e^{\Delta y}}{(k_{1\perp} - q_{1\perp})^2 + k_1 k_2 e^{\Delta y}} + \frac{E}{s} \right) \right\},$$

$$E = (q_{1\perp} - q_{2\perp})(k_{1\perp} - k_{2\perp}) - \frac{1}{\Sigma}(q_1^2 - q_2^2)(k_1^2 - k_2^2) + 2k_1 k_2 \mathrm{sh}(\Delta y) \left(1 - \frac{q_1^2 + q_2^2}{\Sigma} \right).$$

A.2 $gg \to q\bar{q}$

$$\mathcal{A}_{fermions} = N_c^2 \mathcal{A}_{1f} + \mathcal{A}_{2f}$$

$$\begin{aligned}\mathcal{A}_{1f} \;\; = \;\; & \left\{ 2\frac{q_1^2 q_2^2}{s\Sigma}\left(1 + k_1 k_2 \mathrm{sh}(\Delta y)(\frac{1}{t} - \frac{1}{u})\right) - \left(\frac{(k_{1\perp} - q_{1\perp})^2 (k_{2\perp} - q_{1\perp})^2 - k_1^2 k_2^2}{tu}\right)^2 + \right. \\ & \left. \frac{1}{2}\left(\frac{(k_{2\perp} - q_{1\perp})^2 - k_1 k_2 e^{-\Delta y}}{(k_{2\perp} - q_{1\perp})^2 + k_1 k_2 e^{-\Delta y}} - \frac{E}{s}\right)\left(\frac{(k_{1\perp} - q_{1\perp})^2 - k_1 k_2 e^{\Delta y}}{(k_{1\perp} - q_{1\perp})^2 + k_1 k_2 e^{\Delta y}} + \frac{E}{s}\right)\right\} \quad \text{(A.3)}\end{aligned}$$

and

$$\mathcal{A}_{2f} = \left\{ \left(\frac{(k_{1\perp} - q_{1\perp})^2 (k_{2\perp} - q_{1\perp})^2 - k_1^2 k_2^2}{tu}\right)^2 - \frac{q_1^2 q_2^2}{tu} \right\}$$

where E is the same as for gluons.

Appendix B Integrals

To make integration easily it is worth to change integration over Δy to integration over $x = k_1/(k_1 + k_2 e^{\Delta y})$.

$$\int\limits_{-\infty}^{\infty} d\Delta y \, \ldots = \int\limits_0^1 \frac{dx}{x(1-x)} \, \ldots \quad \text{(B.1)}$$

$$\begin{aligned}\int \frac{d^{2+2\varepsilon} k_{2\perp}}{(2\pi)^{2\varepsilon}} \left(-\frac{1}{2}\frac{1}{k_1^2 k_2^2}\right) &= -\frac{1}{2}\int \frac{d^{2+2\varepsilon} k_{2\perp}}{(2\pi)^{2\varepsilon}} \frac{1}{(k_\perp - k_{2\perp})^2 k_{2\perp}^2} \\ &= -\pi \frac{\Gamma(1-\varepsilon)}{(4\pi)^\varepsilon} \frac{k^{2\varepsilon}}{k^2} \frac{\Gamma^2(1+\varepsilon)}{\varepsilon\Gamma(1+2\varepsilon)} \quad \text{(B.2)}\end{aligned}$$

$$\begin{aligned}\int \frac{d^{2+2\varepsilon} k_{2\perp}}{(2\pi)^{2\varepsilon}} \frac{\mathrm{ch}(\Delta y)}{k_1 k_2 s} &= \int \frac{d^{2+2\varepsilon} k_{2\perp}}{(2\pi)^{2\varepsilon}} \left(\frac{1-x}{2k_2^2 s x} + \frac{x}{2k_1^2 s(1-x)}\right) \quad \text{(B.3)} \\ &= \int \frac{d^{2+2\varepsilon} k_{2\perp}}{(2\pi)^{2\varepsilon}} \frac{(1-x)^2}{2k_2^2((1-x)k_\perp - k_{2\perp})^2} + (x \leftrightarrow 1-x) \\ &= \pi \frac{\Gamma(1-\varepsilon)}{(4\pi)^\varepsilon} \frac{k^{2\varepsilon}}{k^2} \frac{\Gamma^2(1+\varepsilon)}{\varepsilon\Gamma(1+2\varepsilon)} [(1-x)^{2\varepsilon} + x^{2\varepsilon}]\end{aligned}$$

Let's now combine these two contributions and perform an integration over x

$$I_{12} = \pi \frac{\Gamma(1-\varepsilon)}{(4\pi)^\varepsilon} \frac{k^{2\varepsilon}}{k^2} \frac{\Gamma^2(1+\varepsilon)}{\varepsilon\Gamma(1+2\varepsilon)} \int \frac{dx}{x(1-x)} [(1-x)^{2\varepsilon} + x^{2\varepsilon} - 1]. \quad \text{(B.4)}$$

In order to avoid divergencies we introduce infinitesimal parameter δ ($\delta \ll \varepsilon$) so that

$$\begin{aligned} \int \frac{dx}{x(1-x)}[(1-x)^{2\varepsilon} + x^{2\varepsilon} - 1] &= \lim_{\delta\to 0} \int \frac{dx}{x^{1-\delta}(1-x)^{1-\delta}}[(1-x)^{2\varepsilon} + x^{2\varepsilon} - 1] \\ = \lim_{\delta\to 0}\left(2\frac{\Gamma(\delta)\Gamma(2\varepsilon+\delta)}{\Gamma(2\varepsilon+2\delta)} - \frac{\Gamma^2(\delta)}{\Gamma(2\delta)}\right) &= \frac{1}{\varepsilon} + 2\psi(1) - 2\psi(1+2\varepsilon) \end{aligned} \tag{B.5}$$

and

$$I_{12} = \pi \frac{\Gamma(1-\varepsilon)}{(4\pi)^\varepsilon} \frac{k^{2\varepsilon}}{k^2} \frac{\Gamma^2(1+\varepsilon)}{\varepsilon\Gamma(1+2\varepsilon)} (\frac{1}{\varepsilon} + 2\psi(1) - 2\psi(1+2\varepsilon)) \tag{B.6}$$

For the integrations involving Σ it is useful to change $k_{2\perp}$ on $\kappa = k_{2\perp} - (1-x)k_\perp$ so that $s = \kappa^2/x(1-x)$ and $\Sigma = \kappa^2/x(1-x) + k^2$. Now

$$\int \frac{dx}{x(1-x)} \int \frac{d^{2+2\varepsilon}k_{2\perp}}{(2\pi)^{2\varepsilon}} \frac{1}{s\Sigma} = \pi \frac{\Gamma(1-\varepsilon)}{(4\pi)^\varepsilon} \frac{k^{2\varepsilon}}{k^2} \frac{\Gamma^2(1+\varepsilon)}{\varepsilon\Gamma(1+2\varepsilon)} \frac{1}{1+2\varepsilon} \tag{B.7}$$

and

$$\int \frac{dx}{x(1-x)} \int \frac{d^{2+2\varepsilon}k_{2\perp}}{(2\pi)^{2\varepsilon}} \frac{E^2}{8q_1^2q_2^2s^2} = \pi \frac{\Gamma(1-\varepsilon)}{(4\pi)^\varepsilon} \frac{k^{2\varepsilon}}{k^2} \frac{\Gamma^2(1+\varepsilon)}{\varepsilon\Gamma(1+2\varepsilon)} \frac{1-\varepsilon}{2(1+2\varepsilon)(3+2\varepsilon)} \tag{B.8}$$

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Figure captions

All cross sections $d\sigma/d^2kdy$ are calculated for combined gluon and quark contributions with $n_f = 4$, $y = 0$

Figure 1 $d\sigma/d^2kdy$ calculated with asymptotic BFKL structure function Eq.(26). $\sqrt{S} = 5.5$TeV, 1-loop α_s with $\Lambda_{QCD} = 200$MeV, $R = 0.7$

Figure 2 $d\sigma/d^2kdy$ calculated with AKMS structure function Eq.(27). $\sqrt{S} = 5.5$TeV, 1-loop α_s with $\Lambda_{QCD} = 200$MeV, $R = 0.7$

Figure 3 $d\sigma/d^2kdy$ calculated with GRV94(NLO) structure function [23]. 2-loop α_s with $\Lambda_{QCD} = 200$MeV, $R = 0.7$
a. $\sqrt{S} = 14$TeV
b. $\sqrt{S} = 5.5$TeV
c. $\sqrt{S} = 1.8$TeV

Figure 4 R-dependence of $d\sigma/d^2kdy$ calculated with GRV94(NLO). $\sqrt{S} = 5.5$TeV, 2-loop α_s with $\Lambda_{QCD} = 200$MeV.

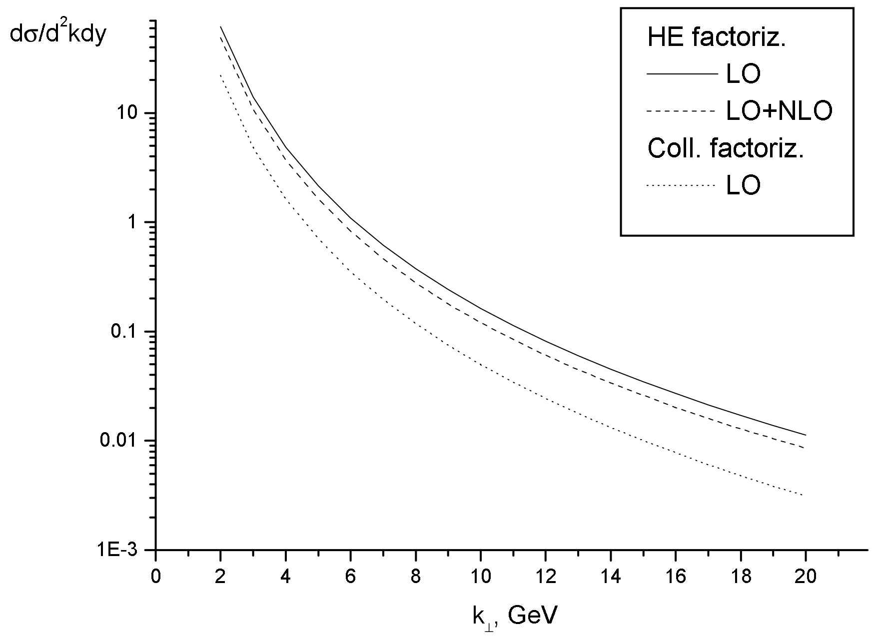


Figure 1

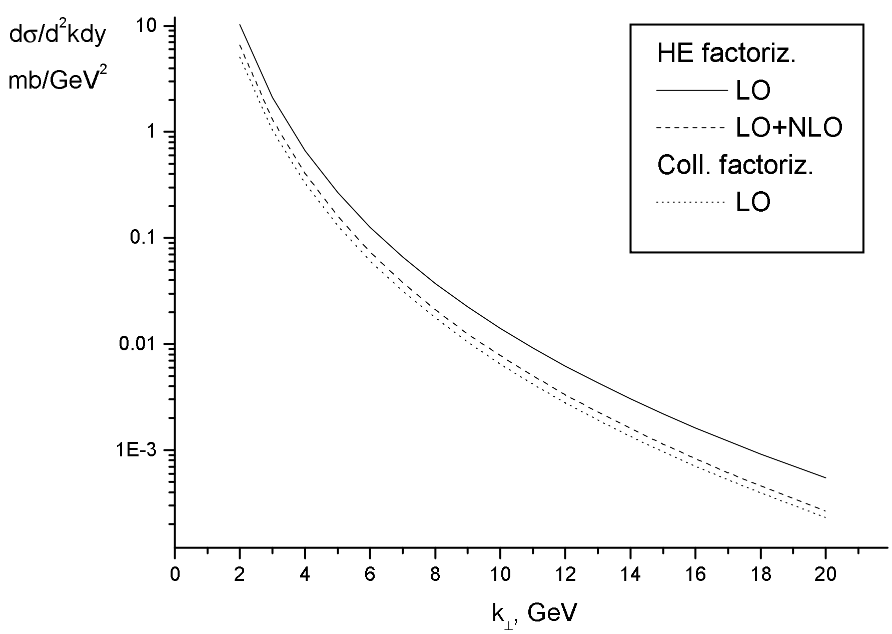
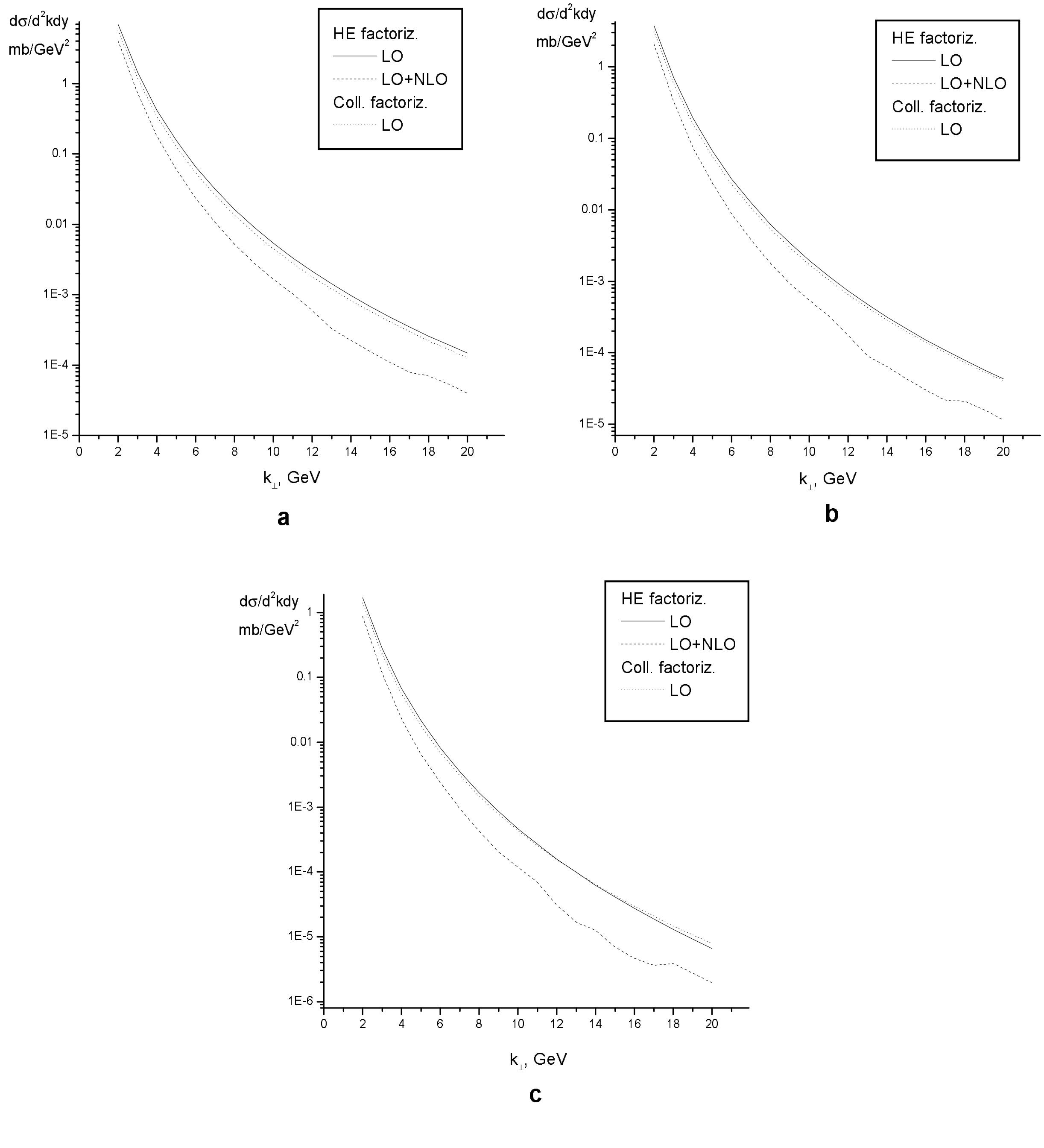


Figure 2



dσ/d²kdy
mb/GeV²
HE factoriz.
LO
LO+NLO
Coll. factoriz.
LO
k⊥, GeV
a
b
c

Figure 3

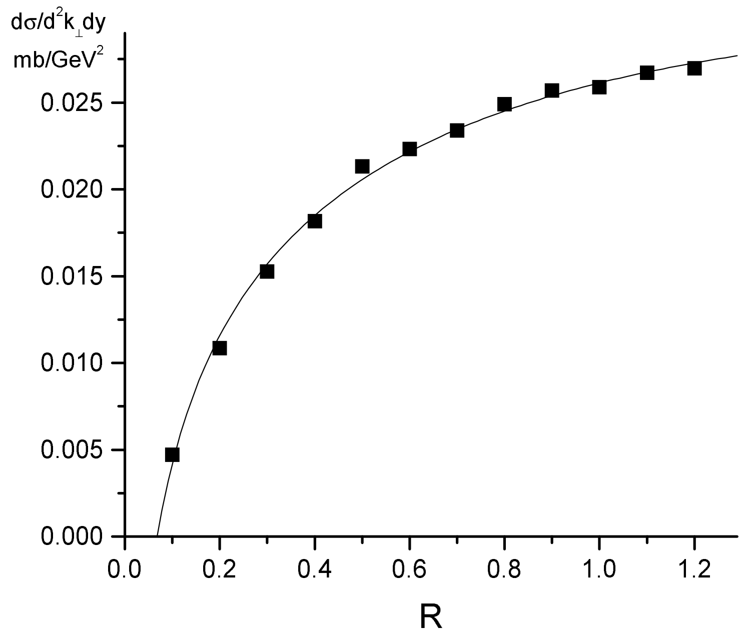

dσ/d²k⊥dy
mb/GeV²
0.025
0.020
0.015
0.010
0.005
0.000
0.0
0.2
0.4
0.6
0.8
1.0
1.2
R

Figure 4